SHEKHAWATI INST. OF ENG.& TECHNOLOGY COLLEGE

SIKAR , RAJASTHAN

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Subject Code & Name: 4AG6 Heat and Mass Transfer

MM: 20 Times: 1.5hrs

MODEL ANSWER PAPER

Q.1 Derive expressions for the conduction through plane wall and composite wall.

One-Dimensional, Steady State Heat Conduction without Heat Generation:

i) Plane Wall or Slab of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a plane wall of thickness ‘L’ and area ‘A’ having uniform conductivity ‘k’ as shown in Figure 1. Temperature on the left hand side of the wall is T_1 and on the right hand side it is T_2. Heat is flowing from left hand side to the right hand side as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+q_g\right]=\rho\, C\frac{\partial T}{\partial t} \qquad (1)$$

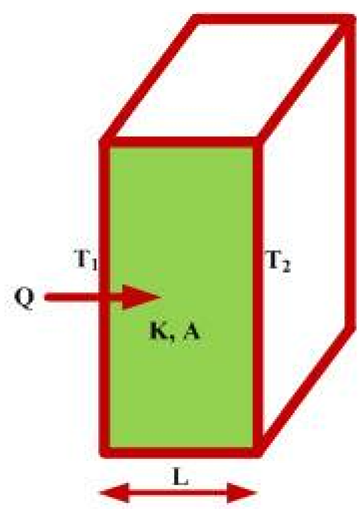

Figure 1

Since it is a case of one-dimensional, stead heat conduction through a wall of uniform conductivity without heat generation, therefore, $\frac{dT}{dt} = 0$, $\frac{dT}{dy} = \frac{dT}{dz} = 0$ and $q_g = 0$ and

Therefore, equation (1) reduces to

$$\frac{d^2T}{dx^2} = 0 \qquad (2)$$

Equation (2) is used to determine the temperature distribution and heat transfer rate through the wall. Integrating equation (2) twice with respect to x, it can written as

$$T = C_1 x + C_2 \qquad (3)$$

Where, C_1 and C_2 are constants of integration.

Using the following boundary conditions:

i. At $x = 0$, $T = T_1$

Equation (3) is written as $C_2 = T_1$ (4)

ii. At $x = L$, $T = T_2$

Equation (3) can be written as $T_2 = C_1 L + C_2$

Or $T_2 = C_1 L + T_1$

$C_1 = (T_2 - T_1)/L$ (5)

Substituting the values of C_1 and C_2 in equation (3)

$$T = \frac{T_2 - T_1}{L} x + T_1$$

Or $$T = T_1 - \frac{T_1 - T_2}{L} x \qquad (6)$$

Equation (6) represents temperature distribution in the wall. It means temperature at any point along the thickness of the wall can be obtained if values of temperatures T_1, T_2, thickness L and distance of the point form either of the faces of the wall are known.

Rate of heat transfer can be determined by using Fourier's law and can be expressed as

$$Q = -kA\frac{dT}{dx} \quad (7)$$

Integrating equation (6) with respect to x to obtain the expression for temperature gradient $\frac{dT}{dx}$

$$\frac{d}{dx}\int T = \frac{d}{dx}\int\left(T_1 - \frac{T_1 - T_2}{L}x\right)$$

$$\frac{dT}{dx} = -\frac{T_1 - T_2}{L}x$$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

Substituting the value of $\frac{dT}{dx}$ from above equation in equation (7), we get

$$Q = -kA\left(\frac{T_2 - T_1}{L}\right) \quad (8)$$

Equation (8) represents the heat transfer rate through the wall.

Composite Slab or Wall:

Consider a composite slab made of three different materials having conductivity k_1, k_2 and k_3, length L_1, L_2 and L_3 as shown in Figure 3. One side of the wall is exposed to a hot fluid having temperature T_f and on the other side is atmospheric air at temperature T_a. Convective heat transfer coefficient between the hot fluid and inside surface of wall is h_i (inside convective heat transfer coefficient) and h_o is the convective heat transfer coefficient between atmospheric air and outside surface of the wall (outside convective heat transfer coefficient). Temperatures at inner and outer surfaces of the composite wall are T_1 and T_4 whereas at the interface of the constituent materials of the slab are T_2 and T_3respectively.

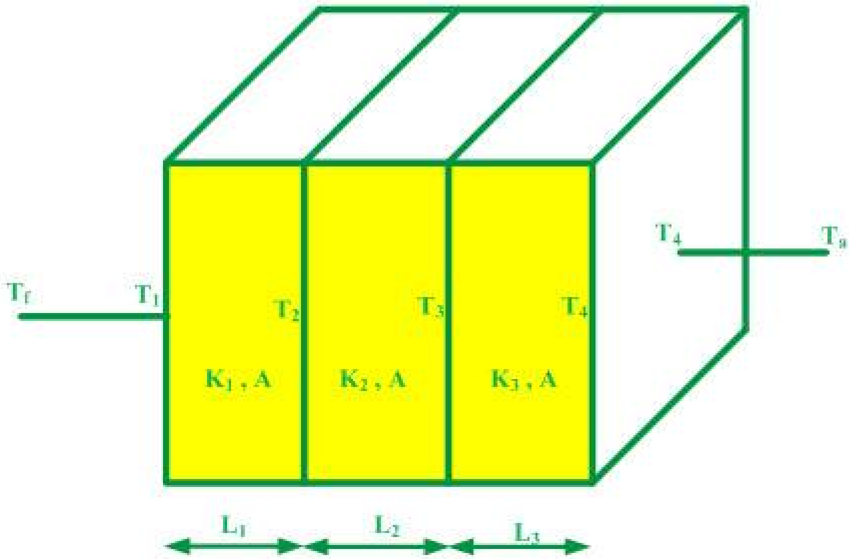

Figure 3

Heat is transferred from hot fluid to atmospheric air and involves following steps:

i) Heat transfer from hot fluid to inside surface of the composite wall by convection

$$Q = h_i\, A\left(T_f - T_1\right)$$

$$\frac{Q}{h_i\, A} = \left(T_f - T_1\right) \tag{36}$$

ii) Heat transfer from inside surface to first interface by conduction

$$Q = \frac{k_1 A\,(T_1 - T_2)}{L_1}$$

$$\frac{Q}{\frac{k_1 A}{L_1}} = (T_1 - T_2) \tag{37}$$

iii) Heat transfer from first interface to second interface by conduction

$$Q = \frac{k_2 A\,(T_2 - T_3)}{L_2}$$

$$\frac{Q}{\frac{k_2 A}{L_2}} = (T_2 - T_3) \tag{38}$$

iv) Heat transfer from second interface to outer surface of the composite wall by conduction

$$Q = \frac{k_3 A\left(T_3 - T_4\right)}{L_3}$$

$$\frac{Q}{\frac{k_3 A}{L_3}} = \left(T_3 - T_4\right) \qquad (39)$$

v) Heat transfer from outer surface of composite wall to atmospheric air by convection

$$Q = h_o\, A\left(T_4 - T_a\right)$$

$$\frac{Q}{h_o\, A} = \left(T_4 - T_a\right) \qquad (40)$$

Adding equations (36), (37), (38) and (40), we get

$$Q\left(\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}\right) = \left(T_f - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_4 + T_4 - T_a\right)$$

or

$$Q\left(\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}\right) = \left(T_f - T_a\right)$$

or

$$Q = \frac{\left(T_f - T_a\right)}{\frac{1}{h_i\, A} + \frac{1}{\frac{k_1 A}{L_1}} + \frac{1}{\frac{k_2 A}{L_2}} + \frac{1}{\frac{k_3 A}{L_3}} + \frac{1}{h_o\, A}} \qquad (41)$$

If composite slab is made of 'n' number of materials, then equation (41) reduces to

$$Q = \frac{\left(T_f - T_a\right)}{\frac{1}{A}\left(\frac{1}{h_i} + \frac{1}{h_o} + \sum_{n=1}^{n=n}\left(\frac{L_n}{k_n}\right)\right)} \qquad (42)$$

If inside and outside convective heat transfer coefficients are not to be considered, then equation (42) is expressed as

$$Q = \frac{(T_f - T_a)}{\frac{1}{A}\left(\sum_{n=1}^{n=n}\left(\frac{L_n}{k_n}\right)\right)} \qquad (43)$$

Q.2 Derive expressions for the conduction through cylindercial wall.

Cylinder of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a cylinder having r_1 and r_2 as inner and outer radii respectively and length 'L' as shown in Figure 2. Temperature of the inner and outer surfaces is T_1 and T_2 respectively. Heat is flowing from inner to outer surface as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k}\right] = \frac{1}{\alpha}\ \frac{\partial T}{\partial t} dt \qquad (9)$$

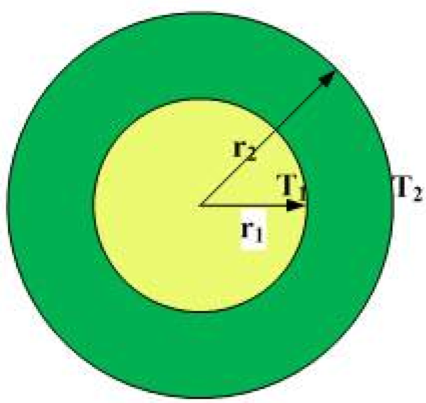

Figure 2

Since it is a case of one-dimensional, stead heat conduction through a wall of uniform conductivity without heat generation, therefore, $\frac{dT}{dt} = 0$, $\frac{dT}{d\phi} = \frac{dT}{dz} = 0$ and $q_g = 0$

Therefore, equation (9) reduces to

$$\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

Or $$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \qquad (10)$$

Equation (10) is used to determine the temperature distribution and heat transfer rate through the cylinder. Integrating equation (10) twice with respect to r, it can written as

$$r\frac{dT}{dr} = c_1 \text{ or } \frac{dT}{dr} = \frac{c_1}{r} \qquad (11)$$

and $T = C_1 \log_e r + C_2$ (12)

Using the following boundary conditions:

i. At $r = r_1$, $T = T_1$

Equation (12) is written as $T_1 = C_1 \log_e r_1 + C_2$ (13)

ii. At $r = r_2$, $T = T_2$

Equation (12) can be written as

$T_2 = C_1 \log_e r_2 + C_2$ (14)

Subtracting equation (14) from equation (13), we get

$T_1 - T_2 = C_1 \log_e r_1 - C_1 \log_e r_2$

$$T_1 - T_2 = C_1 \log_e \frac{r_1}{r_2}$$

$$C_1 = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \qquad (15)$$

Substituting the values of C_1 from equation (15) in equation (13)

$$T_1 = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r_1 + C_2 \qquad (16)$$

$$C_2 = T_1 - \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r_1$$

$$= \frac{T_1 \log_e r_1 - T_1 \log_e r_2 - T_1 \log_e r_1 + T_2 \log_e r_1}{\log_e \frac{r_1}{r_2}}$$

$$= \frac{T_2 \log_e r_1 - T_1 \log_e r_2}{\log_e \frac{r_1}{r_2}}$$

$$C_2 = \frac{T_1 \log_e r_2 - T_2 \log_e r_1}{\log_e \frac{r_2}{r_1}} \qquad (17)$$

Substituting the values of C_1 and C_2 in equation (12), we get

$$T = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \log_e r + \frac{T_1 \log_e r_2 - T_2 \log_e r_1}{\log_e \frac{r_2}{r_1}}$$

$$T = \frac{1}{\log_e \frac{r_2}{r_1}} \left[T_1 \log_e r_2 - T_2 \log_e r_1 - (T_1 - T_2) \log_e r \right]$$

$$T = \frac{1}{\log_e \frac{r_2}{r_1}} \left[T_1 \log_e \frac{r_2}{r} - T_2 \log_e \frac{r}{r_1} \right] \qquad (18)$$

Equation (18) represents temperature distribution in the cylinder. Rate of heat transfer can be determined by using Fourier’s law and can be expressed as

$$Q = \left[-kA\frac{dT}{dr} \right]_{r=r_1} \qquad (19)$$

$$Q = -k \times 2\pi r_1 \times L_1 \left(\frac{dT}{dr} \right)_{r=r_1} \qquad (20)$$

From equation (11) we can write

Substituting the value of C_1 from equation (15), we can write

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r}$$

At $r = r_1$,

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r_1} \qquad (21)$$

Substituting the value of from equation (21) in equation (20), we get

$$Q = -k \times 2\pi r_1 \times L_1 \left(\frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \times \frac{1}{r_1} \right)$$

$$Q = -k \times 2\pi \times L_1 \left(\frac{T_1 - T_2}{\log_e \frac{r_1}{r_2}} \right)$$

$$Q = \frac{2\pi k \times L_1 (T_2 - T_1)}{\log_e \frac{r_2}{r_1}} \qquad (22)$$

Equation (22) represents the heat transfer rate through the cylinder.

Q.3 Derive expressions for the conduction through critical thickness of insulation.

Let us consider a thick insulation layer which is installed around a cylindrical pipe as shown in fig. 3.10 (equivalent electrical circuit is shownin figure 3.11). Let the pipe radius be *R* and the insulation radius is *r*. This (*r*-*R*) will represent the thickness of the insulation. If the fluid carried by the pipe is at a temperature *T* and the ambient temperature is T_a. The insulation of the pipe will alter pipe surface temperature *T* in the radial direction. That is the temperature of the inner surface of the pipe and the outer surface (below insulation) of the pipe will be different. However, if the thermal resistance offered by the pipe is negligible, it can be considered that the temperature (*T*) is same across the pipe wall thickness and it is a common insulation case (please refer previous discussion). It can also be assumed that the heat transfer coefficient inside the pipe is very high as compared to the heat transfer coefficient at the outside of the insulated pipe. Therefore, only two major resistances in series will be available (insulation layer and gas film of the ambient).

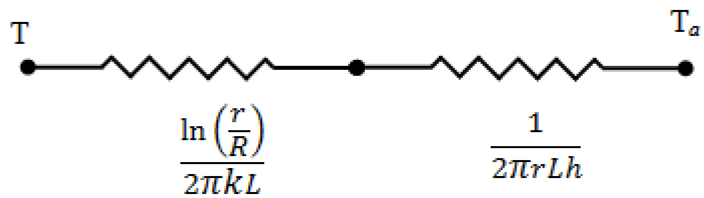

Fig.3.11: Resistance offered by the insulation and ambient gas film

Therefore,

$$\dot{q} = \frac{2\pi L(T - T_a)}{\frac{1}{k}\ln r/R + \frac{1}{rh}}$$

where, k is the thermal conductivity of the material.
On differentiating above equation with respect to *r* will show that the heat dissipation $\dot{q}/L$ reaches a maximum,

$$\frac{d\dot{q}}{dr}\Big|_{r_c} = 2\pi L(T - T_a)\left[\frac{-\left(\frac{1}{k}\right)\left(\frac{1}{\frac{r}{R}}\right)\left(\frac{1}{R}\right)-\left(\frac{1}{h}\right)(-1)(r^{-2})}{\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)^2}\right]\Big|_{r_c}$$

$$= 2\pi L(T - T_a)\left[\frac{\frac{1}{hr^2}-\frac{1}{kr}}{\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)^2}\right]\Big|_{r_c} = 0$$

$$\frac{d^2\dot{q}}{dr^2}\Big|_{r_c} = 2\pi L(T - T_a)\left[\frac{\left\{\left(\frac{1}{h}\right)(-2)\left(\frac{1}{r^3}\right)-\left(\frac{1}{k}\right)(-1)(r^2)\right\} - 2\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)\left\{\frac{1}{k}\left(\frac{1}{\frac{r}{R}}\right)\left(\frac{1}{R}\right)+\left(\frac{1}{h}\right)(-1)(r^{-2})\right\}}{\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)^4}\right]\Big|_{r_c}$$

$$= \frac{2\pi L(T-T_a)\{-\frac{h^2}{k^3}\}}{\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)^4}\Big|_{r_c} < 0$$

So it is maxima, where the insulation radius is equal to

$$r = r_c = \frac{k}{h}$$

where, r_c denotes the critical radius of the insulation. The heat dissipation is maximum at r_c which is the result of the previously mentioned opposing effects.

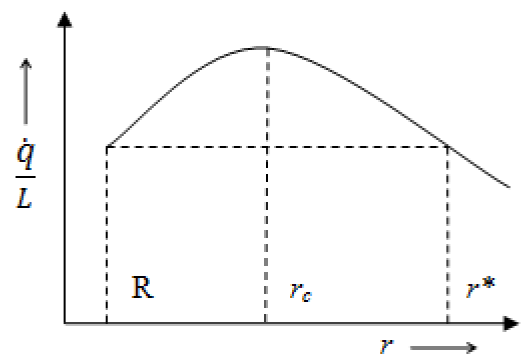

Fig. 3.12: The critical insulation thickness of the pipe insulator

Therefore, the heat dissipation from a pipe increases by the addition of the insulation. However, above r_c the heat dissipation reduces. The same is shown in fig. 3.12.

The careful analysis of the r_c reveals that it is a fixed quantity determined by the thermal properties of the insulator. If $R < r_c$, then the initial addition of insulation will increase the heat loss until $r = r_c$ and after which it begins to decrease. The same heat dissipation which was at bare pipe radius is again attained at r^*. The critical insulation thickness may not always exist for an insulated pipe, if the values of k and h are such that the ratio k/h turns out to be less than R.

It is clear from the above discussion that the insulation above *rc* reduces the heat dissipation from the cylindrical surface. However, if we keep on increasing the insulation the cost of insulation also increases. Thus again there are two opposing factors that must be considered to obtain the optimum thickness. It should be calculated that what is the pay-back period, that is in how many years the cost of insulation is recovered by the cost of energy saving.

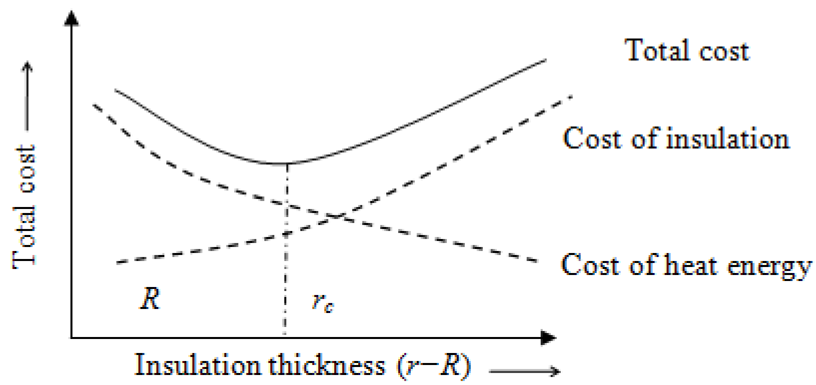

Fig. 3.13: Optimum insulation thickness

The optimum insulation thickness (fig. 3.13) can be determined at which the sum of the insulation cost and the cost of the heat loss is minimum.

Q.4 Derive expressions for the conduction through sphere .

Sphere of Uniform Conductivity without Heat Generation:

Consider steady state heat conduction through a hollow sphere having r_1 and r_2 as inner and outer radii respectively. Temperature of the inner and outer surfaces is T_1 and T_2respectively. Heat is flowing from inner to outer surface as T_1 is greater than T_2. The general conduction equation which governs the conduction heat transfer is written as

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 T}{\partial \phi^2}\right)+\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{q_g}{k}\right]=\frac{1}{\alpha}\ \frac{\partial T}{\partial t} dt \qquad (22)$$

Since it is a case of one-dimensional, steady heat conduction through a shere without heat generation, therefore, $\frac{dT}{dt}=0$, $\frac{dT}{d\phi}=\frac{dT}{d\theta}=0$ and $q_g=0$

Therefore, equation (22) reduces to

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0 \quad (23)$$

Multiplying both sides of equation (23) by r^2 , we get

$$r^2\frac{d^2T}{dr^2} + 2r\frac{dT}{dr} = 0$$

Or

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \quad (24)$$

Equation (24) is used to determine the temperature distribution and heat transfer rate through the wall. Integrating equation (23) twice with respect to r, it can written as

$$r^2\frac{dT}{dr} = c_1 \text{ or } \frac{dT}{dr} = \frac{c_1}{r^2} \quad (25)$$

and

$$T = -\frac{C_1}{r} + C_2 \quad (26)$$

Using the following boundary conditions:

i. At $r = r_1$, $T = T_1$

Equation (26) is written as

$$T_1 = -\frac{C_1}{r_1} + C_2 \quad (27)$$

ii. At $r = r_2$, $T = T_2$

Equation (26) can be written as

$$T_2 = -\frac{C_1}{r_2} + C_2 \quad (28)$$

Subtracting equation (28) from equation (27), we get

$$T_1 - T_2 = C_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$C_1 = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \qquad (29)$$

Substituting the values of C_1 from equation (29) in equation (27)

$$C_2 = T_1 + \frac{1}{r_1}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \qquad (30)$$

Substituting the values of C_1 and C_2 from equations (29) and (30) in equation (26)

$$T = -\frac{1}{r}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} + T_1 + \frac{1}{r_1}\frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$T = T_1 - \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}\left(\frac{1}{r} - \frac{1}{r_1}\right) \qquad (31)$$

Equation (31) represents temperature distribution in a sphere. Rate of heat transfer can be determined by using Fourier's law and can be expressed as

$$Q = \left[-kA\frac{dT}{dr}\right]_{r=r_1} \qquad (32)$$

$$Q = -k \times 4\pi r^2\left(\frac{dT}{dr}\right)_{r=r_1} \qquad (33)$$

From equation (25) we can write

$$\frac{dT}{dr} = \frac{c_1}{r^2}$$

Substituting the value of C_1 from equation (29), we can write

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \times \frac{1}{r^2}$$

At r = r1

$$\frac{dT}{dr} = \frac{T_1 - T_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \times \frac{1}{r_1^2} \qquad (34)$$

Substituting the value of $\frac{dT}{dr}$ from equation (34) from equation (33), we get

$$Q = -k \times 4\pi r^2 \times \frac{T_1 - T_2}{r^2} \frac{1}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$Q = \frac{4\pi r_1 r_2 k (T_1 - T_2)}{(r_2 - r_1)} \qquad (35)$$

Equation (35) represents the heat transfer rate through a sphere.

Q.5 Derive expressions for the multilayer cylindrical wall.

Composite Cylinder:

Consider a composite cylinder consisting of inner and outer cylinders of radii r_1, r_2 and thermal conductivity k_1, k_2 respectively as shown in Figure 4. Length of the composite cylinder is L. Hot fluid at temperature T_f is flowing inside the composite cylinder. Temperature at the inner surface of the composite cylinder exposed to hot fluid is T_1 and outer surface of the composite cylinder is at temperature T_3 and is exposed to atmospheric air at temperature T_a. The interface temperature of the composite cylinder is T_2.. Convective heat transfer coefficient between the hot fluid and inside surface of composite cylinder is h_i (inside convective heat transfer coefficient) and h_o is the convective heat transfer coefficient between atmospheric air and outside surface of the composite cylinder (outside convective heat transfer coefficient). Heat is transferred from hot fluid to atmospheric air and involves following steps:

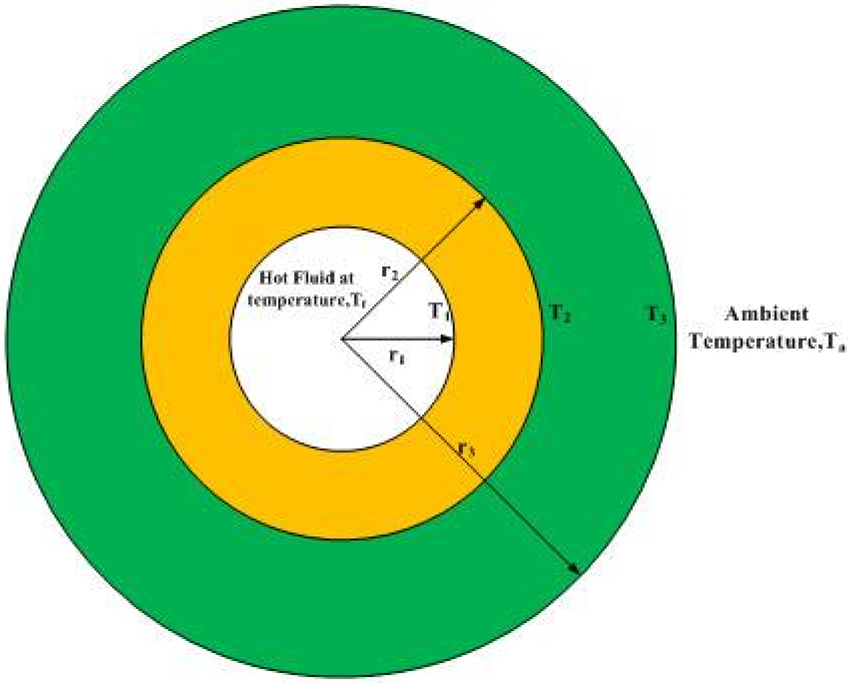

Figure 4

i) Heat transfer from hot fluid to inside surface of the composite cylinder by convection

$$Q = h_i A\left(T_f - T_1\right)$$

$$Q = h_i 2\pi r_1 L\left(T_f - T_1\right)$$

$$\frac{Q}{h_i 2\pi r_1 L} = \left(T_f - T_1\right) \quad (44)$$

ii) Heat transfer from inside surface to interface by conduction

$$Q = \frac{k_1 2\pi L\left(T_1 - T_2\right)}{\log_e \frac{r_2}{r_1}}$$

$$\frac{Q}{\frac{k_1 2\pi L}{\log_e \frac{r_2}{r_1}}} = \left(T_1 - T_2\right) \quad (45)$$

iii) Heat transfer from interface to outer surface of the composite cylinder by conduction

$$Q = \frac{k_2 2\pi L\left(T_2 - T_3\right)}{\log_e \frac{r_3}{r_2}}$$

$$\frac{Q}{\frac{k_2 2\pi L}{\log_e \frac{r_3}{r_2}}} = \left(T_2 - T_3\right) \qquad (46)$$

iv) Heat transfer from outer surface of composite wall to atmospheric air by convection

$$Q = h_o\, 2\pi r_3 L\left(T_4 - T_a\right)$$

$$\frac{Q}{h_o\, 2\pi r_3 L} = \left(T_4 - T_a\right) \qquad (47)$$

Adding both sides of equations (44), (45),(46) and (47), we get

$$\frac{Q}{2\pi L}\left(\frac{1}{h_i r_1}+\frac{1}{\frac{k_1}{Log_e \frac{r_2}{r_1}}}+\frac{1}{\frac{k_2}{Log_e \frac{r_3}{r_2}}}+\frac{1}{h_o r_3}\right)=\left(T_f - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_a\right)$$

or

$$Q\left(\frac{1}{h_i r_1}+\frac{1}{\frac{k_1}{Log_e \frac{r_2}{r_1}}}+\frac{1}{\frac{k_2}{Log_e \frac{r_3}{r_2}}}+\frac{1}{h_o r_3}\right)=2\pi L\left(T_f - T_a\right)$$

or

$$Q=\frac{2\pi L\left(T_f - T_a\right)}{\left(\frac{1}{h_i r_1}+\frac{1}{\frac{k_1}{Log_e \frac{r_2}{r_1}}}+\frac{1}{\frac{k_2}{Log_e \frac{r_3}{r_2}}}+\frac{1}{h_o r_3}\right)} \qquad (48)$$

If the composite cylinder consists of 'n' cylinders, then equation (48) can be expressed as:

$$Q=\frac{2\pi L\left(T_f - T_a\right)}{\left(\frac{1}{h_i r_1}+\frac{1}{h_o r_{n+1}}+\sum_{n=1}^{n=n}\frac{1}{k_n}Log_e\left(\frac{r_{n+1}}{r_n}\right)\right)} \qquad (49)$$

If inside and outside convective heat transfer coefficients are not to be considered, then equation (3.41) is expressed as

$$Q=\frac{2\pi L\left(T_f - T_a\right)}{\left(\sum_{n=1}^{n=n}\frac{1}{k_n}Log_e\left(\frac{r_{n+1}}{r_n}\right)\right)} \qquad (50)$$