SHEKHAWATI INSTITUTE OF ENGINEERING & TECHNOLOGY, SIKAR, (RAJASTHAN)

I Midterm Exam 2017-18 (B. Tech. IV SEM CS)

Subject code & Name: 4CS2A & DMS

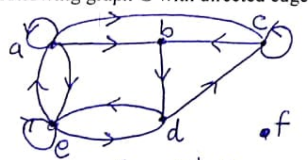
MM: 20 Time: 1:30Hr

Instructions for Students

1. **Attempt any five questions.**
2. Don't roll or tear answer sheet.
3. Write your ID before starting question paper.

Q.1 Draw a graph which is Eulerian as well as Hamiltonian.

Q.2 Find the in degree and out degree of each vertex in the following graph G with directed edges.



Graph G

Q.3 Sketch the graph with adjacency matrix

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

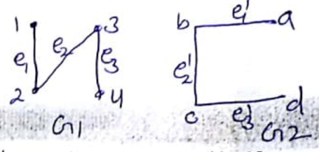
Order of vertices are a,b,c,d.

Q.4 Explain the following terms:

(i) Regular graph and Bipartite graph.

(ii Walk, Trail, Path and circuit

Q.5 Prove that the graphs G1 and G2 are isomorphic.



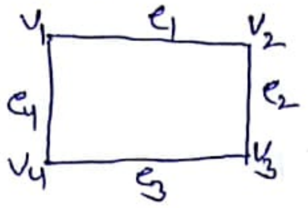
Q.6 A computer company must hire 25 programmers to handle system programming jobs and 40 for the application programming .Of the hired persons, 10 will have to do the jobs of both types, Find how many programmers must be hired?

Q.7 Let A={a,b,c,d,e} and B={c,e,f,h,k,m} then prove if A and B are finite sets then |AUB|= |A|+|B|-|A B|.

I^st Mid term Exam. 2018 B.Tech IV^th sem (CS)
Mid term Paper Solution

Q.1 Draw a graph which is ~~eb~~ eularian as well as hamiltonian

Ans

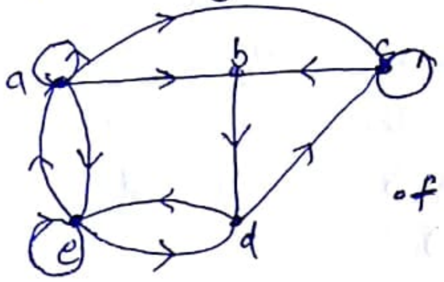


Graph G

let $w = v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_1$ be a closed walk. In this walk w each edge is traversed only once.

Therefore it is an Eulerian graph. The walk w contains all vertices of G and no vertex and no. edge is repeated in. w. So it is a closed Hamiltonian path. So we can say the graph is both Eulerian & Hamiltonian

Q.2. Find the indegree and outdegree of each vertex in the following graph G with directed edges.



Graph G

Ans The Indegrees in Graph G are:

$deg^-(a) = 2$, $deg^-(e) = 3$ and
$deg^-(b) = 2$, $deg^-(f) = 0$
$deg^-(c) = 3$
$deg^-(d) = 2$

The outdegrees are:

$deg^+(a) = 4$, $deg^+(b) = 1$, $deg^+(c) = 2$, $deg^+(d) = 2$,
$deg^+(e) = 3$ and $deg^+(f) = 0$

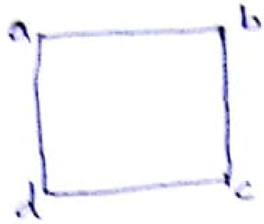
Q.3. Sketch the graph with adjacency matrix.

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

with respect to the ordering of vertices a, b, c, d.

P.T.O.

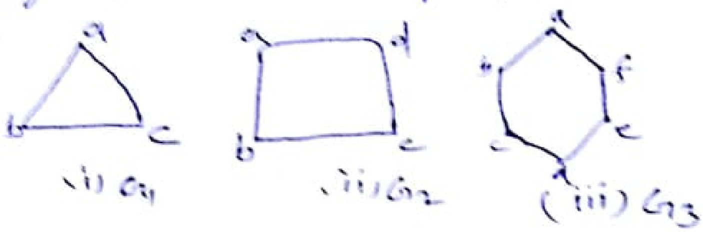
Ans. A Graph with the given adjacency matrix is : ②



Q.4. Explain the following terms :-

(i) Regular graph & Bipartite graph

Ans. Regular graph - If every vertex of the graph has the same degree then that graph is called regular graph. eg -



Bipartite graph :- A simple graph G is called a bipartite if its vertex set V can be partitioned into two disjoint nonempty sets V_1 and V_2 such that every edge in graph connects a vertex in V_1 and a vertex in V_2. (so that no edge in G connects either two vertices in V_1 or two vertices in V_2)

4.(ii) Walk, Trail, Path and circuit

Ans: Walk - In a graph $G=(V,E)$, a sequence of vertices and edges, denoted by W.

$$W = v_0 e_1 v_1 e_2 v_2 \ldots v_{n-1} e_n v_n \quad (n \geq 0)$$

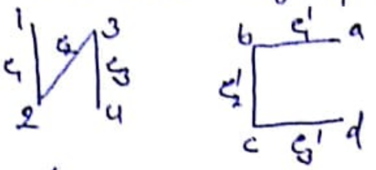
A walk starting from vertex v_0 and ending with vertex v_n is called $v_0 - v_n$ walk.

Trail - An open walk in which no edge is repeated is called a trail.

Path - An open walk in which no vertex is repeated is called a path.

Circuit - A closed walk in which no edge is repeated is called a circuit.

Q.5. Prove that the graph G_1 and G_2 are isomorphic.



G_1 G_2

Ans: In the above graph G_1 & G_2 the number of vertices of G_1 = number of vertices of $G_2 = 4$ and number of edges of G_1 = number of edges in $G_2 = 3$.

Now the mapping is -

$f: G_1 \to G_2$, by $f(1) = a$, $f(2) = b$, $f(3) = c$ & $f(4) = d$

and $(1,2) = e_1 \in E(G_1) \Rightarrow (f(1), f(2)) = (a, b) = e_1' \in E(G_2)$

$(2,3) = e_2 \in E(G_1) \Rightarrow (f(2), f(3)) = (b, c) = e_2' \in E(G_2)$

$(3,4) = e_3 \in E(G_1) \Rightarrow (f(3), f(4)) = (c, d) = e_3' \in E(G_2)$

$(1,4) \notin E(G_1) \Rightarrow (f(1), f(4)) = (a, d) \notin E(G_2)$

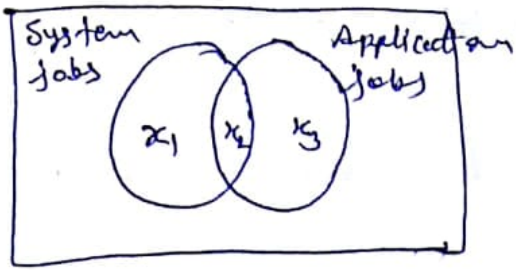
$(2,4) \notin E(G_1) \Rightarrow (f(2), f(4)) = (b, d) \notin E(G_2)$ and

$(1,3) \notin E(G_1) \Rightarrow (f(1), f(3)) = (a, c) \notin E(G_2)$.

Therefore f is one-one, onto, and preserves adjacency as well as non adjacency of vertices. So we can say G_1 and G_2 are isomorphic to each other.

Q.6 A computer company must hire 25 programmers to handle system programming jobs and 40 for the application programming. Of the hired persons, 10 will have to do the jobs of both types. Find how many programmers must be hired?

Ans



$x_1 + x_2 = 25$

$x_2 + x_3 = 40$

$x_2 = 10$

So, $x_1 = 25 - 10 = 15$

$x_3 = 40 - 10 = 30$

Hence $x_1 + x_2 + x_3 = 15 + 10 + 30 = 55$

Q.7 Let $A = \{a, b, c, d, e\}$ and $B = \{c, e, f, h, k, m\}$ then prove if A and B are finite sets then $|A \cup B| = |A| + |B| - |A \cap B|$.

Ans. $A = \{a, b, c, d, e\}$ and $B = \{c, e, f, h, k, m\}$ are given.

Then $A \cup B = \{a, b, c, d, e, f, h, k, m\}$ and

$A \cap B = \{c, e\}$

$\therefore |A| = 5, |B| = 6, |A \cup B| = 9, |A \cap B| = 2$

L.H.S $= \cancel{|A A} \; |A \cup B| = 9$

R.H.S $= |A| + |B| - |A \cap B| = 5 + 6 - 2 = 9$

$\therefore$ L.H.S. = R.H.S

Hence $|A \cup B| = |A| + |B| - |A \cap B|$.