SHEKHAWATI INSTITUTE OF ENGINEERING & TECHNOLOGY

SIKAR , RAJASTHAN

1st MID TERM EXAMINATION 2017-18 (B.Tech 4th sem CS)

Subject Code & Name: SPT

MM: 20 Time: 2Hr

Instructions for Students

1. Use pencil for diagrams.
2. Answer should mark proper S No.
3. Don't role or tear answer sheet.
4. Write your ID before starting question paper.

Attempt all four questions.

Q.1 Fit a straight line using the following data.

x	1	2	3	4	5
y	7	15	20	25	34

Q.2 Find the correlation coefficient using by rank method.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Q.3 Show that the α, the acute angle between the two lines of regression is given by :

$$\tan \alpha = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

Q.4 In a bolt factory machines A , B and C manufacture respectively 25 % , 35 % and 40 % of the total. Of their output 5 , 4 , 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What the the probabilities that it was manufactured by machine A, b or C.

Q.5 An anti aircraft gun can take four shots at enemy plane moving away from it. The probability of hitting the plane at the 1st , 2nd , 3rd and 4th shots are 0.4 , 0.3 , 0.2 and 0.1 respectively. Then find the chance that the gun hits the plane.

Q.6. If 10% of the bolts produced by a machine are defective. Then find the probability that out of 5 bolts are selected at random then atmost one will be defective.

Q. 7. A random variable X has following probability distributions.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2 + k$

I. Find k.

II. Evaluate $P(X<6)$, $P(X\geq6)$ and $P(0<X<5)$.

III. Find distribution function of X.

SPT

Q.1) Fit a straight line using the following data.

Ans

x	y	x^2	xy
1	7	1	7
2	15	4	30
3	20	9	60
4	25	16	100
5	34	25	170
$\Sigma x = 15$	$\Sigma y = 101$	$\Sigma x^2 = 55$	$\Sigma xy = 367$

$n = 5.$

Let the straight line to be fitted is.

$$y = a + bx \quad —(1)$$

then the subsidary eq^n are.

$$\Sigma y = \Sigma a + b\Sigma x$$

$$\Rightarrow \Sigma y = na + b\Sigma x \quad —(2)$$

$$\& \quad \Sigma xy = a\Sigma x + b\Sigma x^2 \quad —(3)$$

from eq^n (2) & (3)

$$5a + 15b = 101 \quad —(4)$$

$$15a + 55b = 367 \quad —(5)$$

then multiplying eq^n (4) by 3. & evalute.

$$\cancel{15a} + 55b = 367$$

$$\cancel{15a} + 45b = 303$$

$$\overline{}$$

$$10b = 64$$

$$\boxed{b = 6.4}$$

then put in eq^n (4).

$$5a + 15 \times 6.4 = 101$$

$$5a + 96 = 101$$

$$5a = \frac{101 - 96}{5}$$

$$\boxed{a = 1}$$

So the required line will be.

$$\boxed{y = 1 + 6.4x}$$

Q82) Find the correlation cofficient using by rank method.

x	y	xi	yi	di=xi-yi	di²
1	9	9	8	1	1
2	8	8	9	-1	1
3	10	7	7	0	0
4	12	6	5	1	1
5	11	5	6	-1	1
6	13	4	4	0	0
7	14	3	3	0	0
8	16	2	1	1	1
9	15	1	2	-1	1
					Σd_i^2 6.

Sol:.

$$\rho = 1 - \frac{6\Sigma d_i^2}{n(n^2-1)}$$

$$\rho = 1 - \frac{6\times 6}{9(9^2-1)}$$

$$\rho = 1 - \frac{36}{9(81-1)}$$

$$\rho = 1 - \frac{36}{9(80)}$$

$$\rho = 1 - \frac{36}{720}$$

$$\rho = \frac{720-36}{720}$$

$$\rho = \frac{684}{720}$$

$$\boxed{\rho = 0.95}$$

Q.3). Show that the α, the acute angle between the two lines of regression is given by. (3)

$$\tan\alpha = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

Ans. The lines of regression on y on x is given by

$$(y-\bar{y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \quad \text{—— (1)}$$

Line of regression on x on y is given by

$$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y}) \quad \text{—— (2)}$$

then. Slope of eq^n (1) is

$$d_{yx} = \frac{r\sigma_y}{\sigma_x}$$

& Slope of eq^n. (2) is

$$d_{xy} = \frac{1}{r}\frac{\sigma_y}{\sigma_x}$$

hence we know that $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$$\tan\theta = \left|\frac{r\frac{\sigma_y}{\sigma_x} - \frac{1}{r}\frac{\sigma_y}{\sigma_x}}{1+\frac{r\sigma_y}{\sigma_x}\frac{1}{r}\frac{\sigma_y}{\sigma_x}}\right|$$

$$\tan\theta = \left|\frac{\sigma_x\sigma_y(r^2-1)}{r(\sigma_x^2+\sigma_y^2)}\right|$$

$$\tan\alpha = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

Q:4] In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. of their output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What the probablities that is was manufactured by machine A, b, or C.

Ans. Let B_1 is the event that the Bolt manufacturd by machine A

Let B_2 is the event that the Bolt manufactured by machine B

Let B_3 is the event that the Bolt manufactured by machine C.

& Let A be the event that the Bolt. is deffective.

$$P(B_1) = \frac{25}{100} = \frac{1}{4}$$

$$P(B_2) = \frac{35}{100} = 7/20$$

$$P(B_3) = 40/100 = 2/5$$

& $P(A/B_1) = 5/100$

$P(A/B_2) = 4/100$

& $P(A/B_3) = 2/100$

from byies theorem we know that

$$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{P(A)}$$

where $P(A) = \sum P(B_i) \cdot P(A/B_i)$

$$P(A) = \frac{1}{4} \cdot \frac{5}{100} + 7/20 \times \frac{4}{100} + \frac{2}{5} \times \frac{2}{100}$$

$$P(A) = \frac{1}{80} + \frac{7}{500} + 2/250$$

$$P(A) = 69/2000$$

Now the probablity that the bolt is deffective and it is from machine A

$$P(B_1/A) = \frac{P(B_1) \cdot P(A/B_1)}{P(A)}$$

$$P(B_1/A) = \frac{\frac{1}{4} \cdot 5/100}{69/2000} = \frac{5 \times 20}{4 \times 69} = \frac{100}{276}$$

$$= 0.3623$$

$$P(B_2/A) = \frac{P(B_2) \cdot P(A/B_2)}{P(A)}.$$

$$P(B_2/A) = \frac{\frac{7}{20} \times \frac{4}{100}}{69/2000} = \frac{28 \times 2}{2 \times 69} = \frac{56}{138} = 0.40$$

& $$P(B_3/A) = \frac{P(B_3) \cdot P(A/B_3)}{P(A)}.$$

$$= \frac{2/5 \times 2/100}{69/2000} = \frac{4 \times 4}{69} = \frac{16}{69} = 0.231$$

Q:5) An anti air craft gun can take four shots at an enemy plane moving away from it. the Probablity of hiting the plane at the 0,1,2 3&4 shot are 0.4, 0.3, 0.2 & 0.1 respectively.

find the chance that the gun hits the plane

Sol

Let the events of hitting the plan at 1, 2, 3, & 4. Chance or S_1, S_2, S_3 & S_4 respectively.

$$P(S_1) = 0.4$$
$$P(S_2) = 0.3$$

$$P(S_3) = 0.2$$
$$P(S_4) = 0.1$$

Probablity of hitting the Plan

$$= P(S_1) + P(\bar{S_1})\, P(S_2) + P(\bar{S_1})\, P(\bar{S_2})\, P(S_3) + P(\bar{S_1})\, P(\bar{S_2})\, P(\bar{S_3})\, P(S_4)$$

$$= (0.4) + (0.6)\times(0.3) + (0.6)(0.7)(0.2) + (0.6)(0.7)(0.8)(0.1)$$

$$= 0.4 + 0.18 + 0.084 + 0.0336$$

$$= 0.6976$$

Q6 If 10% of the bolts produced by a machine are defective find the probability that out of 5 bolts selected at random, at the most one will be defective?

Solution:- Given that

Probability of defective bolts $s = p = 10\% = \frac{1}{10}$

$\therefore$ Probability of non-defective bolts $= q = 1 - \frac{1}{10} = \frac{9}{10}$

Also given that number of bolts $n = 5$

$\therefore$ Probability that at the most one defective

$= P(\text{no bolt is defective}) + P(\text{one bolt of defective})$

$= 5_{C_0} p^0 q^5 + 5_{C_1} p^1 q^4 = \left(\frac{9}{10}\right)^5 + 5 \cdot \frac{1}{10}\left(\frac{9}{10}\right)^4$

$= \frac{9^4}{10^5}(9+5) = \frac{6561}{10^5} \times 14$

$= 0.9184$

Q7 A random variable X has following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(i) find K

(ii) Evaluate $P(X<6)$, $P(X\geq 6)$ and $P(0<X<5)$

(iii) find distribution function of X.

Solution:- (i) clearly X is a discrete random variable and probability distribution of X is given

we have $\sum_{i=0}^{7} P_i = 1 \Rightarrow 10K^2 + 9K - 1 = 0$

$\Rightarrow (K+1)(10K-1) = 0$

$\Rightarrow K = -1, \frac{1}{10}$

$-$ve value of K is not possible because $P_i \geq 0$

(ii) $P(X<6) = \sum_{i=0}^{5} P_i = \sum_{i=0}^{7} P_i - (P_6+P_7)$

$= P_0+P_1+P_2+P_3+P_4+P_5 = 1-P(X\geq 6)$

$= 1-[P(X=6)+P(X=7)]$

$= 1-(9K^2+K) = 1-\frac{1}{10}-\frac{9}{100} = \frac{81}{100}$

Now $P(X\geq 6) = 1-P(X<6) = 1-\frac{81}{100} = \frac{19}{100}$

Now $P(0<X<5) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 8K = \frac{8}{10} = \frac{4}{5}$

(iii) Now distribution function of random variable X

$$F(x) = P(X\leq x) = \sum_{j=0}^{x} P(X=j) = \begin{cases} 0 & ; x=0 \\ 1/10 & ; x\leq 1 \\ 3/10 & ; x\leq 2 \\ 5/10 & ; x\leq 3 \\ 8/10 & ; x\leq 4 \\ 81/100 & ; x\leq 5 \\ 83/100 & ; x\leq 6 \\ 1 & ; x\leq 7 \end{cases}$$